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RADIATION (ABSORBING) BOUNDARY CONDITIONS* FOR ELECTROMAGNETIC FIELDS

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Introduction

An important problem in finite difference or finite element computation of the electromagnetic field obeying the space-time Maxwell equations with self-consistent sources is that of truncating the outer numerical boundaries properly to avoid spurious numerical reflection. Methods for extrapolating properly the fields just beyond a numerical boundary in free space have been treated by a number of workers. Lindman[1] considered reflection of both propagating and evanescent plane waves at various angles of incidence to a boundary. He used projection operators which process past data at the boundary to update three to six wave equations there. Engquist and Majda[2] developed a systematic method for "manipulating symbols" to obtain a hierarchy of local boundary conditions at the artificial (truncating) boundaries. They considered 2-dimensional waves impinging on plane boundaries from various directions. Holland[3] described a radiation boundary condition for the field scattered from a 3-dimensional object based on an r^{-1} behavior. He observed that a good empirical estimate of "sufficiently large r" was about d/2 beyond the scatterer in every direction, d being its largest dimension. This fact motivates our effort to obtain radiation boundary conditions accurate to order $(|\bar{r}_{source}|_{max}/r)^2$. Mur[4], motivated by the work of Engquist and Majda, manipulated the 3-dimensional scalar wave equation for a radiated field component into a form appropriate for waves at various angles of incidence to a planar boundary. He obtained a second-order finite difference equation which proved to be very efficient for extrapolating tangential $\bar{E}$ just beyond the boundary.

We intend to avoid plane wave assumptions and derive boundary conditions more directly related to the source distribution within the region. We use the Panofsky-Phillips' relations,[5] which enable one to extrapolate conveniently the vector field components parallel ($\parallel$) and perpendicular ($\perp$) to a radial from the coordinate origin chosen near the center of the charge-current distribution.

Analysis

The Panofsky-Phillips equations describe the space-time fields

$$4\pi\sqrt{\varepsilon_0/\mu_0}\,\bar{E}(\bar{r},t) \equiv \bar{e}(\bar{r},t) = \bar{e}_1(\bar{r},t) + \bar{e}_2(\bar{r},t) \tag{1}$$

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$$4\pi\, \bar{H}(r,t) = \bar{h}(\bar{r},t) \qquad (2)$$

according to volume integrals over retarded sources (in brackets):

$$\bar{e}_1(\bar{r},t) = c \int \frac{[\rho(\bar{r}',t')]\bar{R}(\bar{r},\bar{r}')}{R^3}\,dv' + \int \frac{[J(\bar{r}',t')]\cdot\bar{R}\bar{R}}{R^4}\,dv' \qquad (3)$$

$$\bar{e}_2(\bar{r},t) = \int \frac{([J]\times\bar{R})\times\bar{R}}{R^4}\,dv' + \frac{1}{c}\int \frac{([\dot{J}]\times\bar{R})\times\bar{R}}{R^3}\,dv' \qquad (4)$$

$$\bar{h}(\bar{r},t) = \int \frac{([J]\times\bar{R}}{R^3}\,dv' + \frac{1}{c}\int \frac{[\dot{J}]\times\bar{R}}{R^2}\,dv' \qquad (5)$$

Here the retarded time t' from a retarded source $[\rho]$, $[J]$, or $[\dot{J}]$ at $\bar{r}'$ contributing field to the observative space-time point $(\bar{r},t)$ is

$$t' = t - R/c \quad , \quad R = |\bar{R}| = |\bar{r}-\bar{r}'| \quad . \qquad (6)$$

The dot denotes $\partial/\partial t'$ at the source or $\partial/\partial t$ at the observation time.

We have applied these equations to extrapolate the fields at point $(\bar{r}+\bar{dr},\ t+dt)$ just beyond the numerical boundary in Fig. 1 from those fields at $(\bar{r},t)$ on the boundary. $\bar{r}$ is a vector from an origin chosen to minimize $|\bar{r}'_{max}/\bar{r}|^2$. $\bar{dr} = \bar{a}_{\parallel}dr$ is chosen parallel to $\bar{r}$; components such as $\bar{E}_{\perp}$ are perpendicular to $\bar{r}$. The $\bar{e}_1$-portion of $\bar{e}$, consisting of "near-field" in (3), is primarily longitudinal (i.e., parallel to $\bar{r}$) but has a small transverse component ($\perp\bar{r}$) to be retained. The $\bar{e}_2$- and $\bar{h}$- fields, each of which contains a near-field component $\propto [J]$ and a "radiation" field component $\propto [\dot{J}]$, are primarily transverse but have small longitudinal components to be retained.

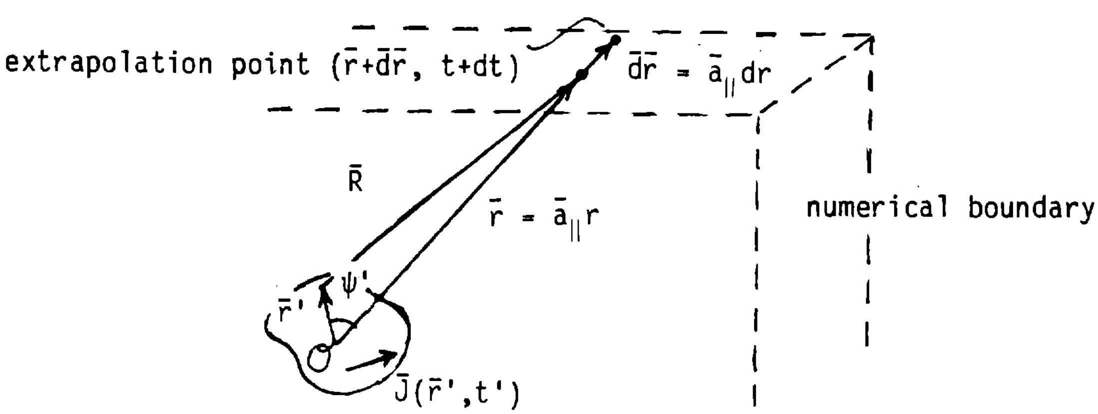

Fig. 1. Geometry for deriving the fields extrapolated to point $(\bar{r}+\bar{dr},\ t+dt)$ from those at $(\bar{r},t)$ from the Panofsky-Phillips relations.

A very convenient property for analysis with $\bar{r}$ fixed is that an arbitrary scalar source $[\rho(\bar{r}',t' = t-R/c)]$ or vector $[J(\bar{r}',t')]$ is unchanged within second order of (r'/r) if $(\bar{r},t)$ is changed to $(\bar{r}+\bar{d}\bar{r}, t+dt)$, provided $dt = dr/c$. This implies

$$t' = t - \frac{R(\bar{r}',\bar{r})}{c} = t+dt - R_+(\bar{r}',\bar{r}+\bar{d}\bar{r})/c \quad , \quad dt = dr/c \tag{7}$$

and follows easily from an expansion of $R(\bar{r}',\bar{r})$:

$$R(\bar{r}',\bar{r}) = [r^2+(r')^2-2rr'\cos\psi']^{1/2} \approx r[1 - \frac{r'}{r}\cos\psi' + \mathcal{O}(\frac{r'}{r})^2] \tag{8}$$

Thus

$$R_+(\bar{r}', \bar{r}+\bar{d}r) \approx r+dr - r'\cos\psi' + \mathcal{O}(\frac{r'}{r})^2 \tag{9}$$

and (7) is verified to second-order. Therefore, we may compute changes in the fields (3)-(5) from $(\bar{r},t)$ to $(\bar{r}+\bar{d}r, t+dt)$ <u>without changing</u> $[\rho]$, $[J]$, or $[\dot{J}]$ in the integrands!

The results of computing the changes in the fields within second order of (r'/r) are summarized as follows:

$$(\bar{e})_{\parallel}(\bar{r}+\bar{d}r, t+dt) - (\bar{e})_{\parallel}(\bar{r},t) = d(\bar{e})_{\parallel} = -2\,\frac{dr}{r}\,(\bar{e}(\bar{r},t))_{\parallel} \tag{10}$$

$$d(\bar{e}_1)_{\perp} = -3\,\frac{dr}{r}\,(\bar{e}_1(\bar{r},t))_{\perp} \tag{11}$$

$$d(\bar{e}_2)_{\perp} = d(\bar{h})_{\perp} \times \bar{a}_{\parallel} \tag{12}$$

$$d(\bar{h})_{\parallel} = -2\,\frac{dr}{r}\,(\bar{h}(\bar{r},t))_{\parallel} \tag{13}$$

These depend on the approximations

$$\bar{d}\bar{R} = \bar{d}\bar{r} \approx \bar{R}dr/r \approx \bar{R}_{\parallel}dr/r \quad , \tag{14}$$

which are made when second-order accuracy can be retained, and

$$\omega\, R(\bar{r}',\bar{r})/c > 1 \quad , \tag{15}$$

at all significant frequencies in the sources. This implies the numerical errors will be larger at the <u>lower</u> frequencies.

Equations (10)-(13) suggest a procedure for advancing the field components from $(\bar{r},t)$ to $(\bar{r}+\bar{d}r, t+dt)$:

A) Advance $\bar{e}_{\parallel}(\bar{r},t)$ by $d(\bar{e})_{\parallel}$ according to (10).

B) Compare the fields at $(\bar{r}-\bar{dr},\ t-dt)$ with those at $(\bar{r},t)$ to obtain the total $d(\bar{e})_{\perp} = d(\bar{e}_1)_{\perp} + d(\bar{e}_2)_{\perp}$ and $d(\bar{h})_{\perp}$. Use (12) to find $d(\bar{e}_2)_{\perp}$ and then $d(\bar{e}_1)_{\perp}$.

C) Then use (11) to find $(\bar{e}_1(\bar{r},t))_{\perp}$, and then $(\bar{e}_2(\bar{r},t))_{\perp}$.

D) With the $(\bar{e})_{\perp}$-field thus separated at $(\bar{r},t)$, proceed to advance $(\bar{e}_1)_{\perp}$ and $(\bar{e}_2)_{\perp}$ to $\bar{r}+\bar{dr}$, t+dt.

E) If desired, advance $(\bar{h})_{\parallel}$ by (13).

Implementation

We intend to implement this procedure in the finite-difference time-domain code GFDTD for sources within the numerical rectangular parallelopiped in Fig. 1. Since the extrapolation equations are referred to an $\bar{r}$ vector which is generally oblique to the boundary, we must project the field changes--in particular, those of $\bar{e}$--to a grid of rectangular cells on the boundary. Once the $\bar{e}$-fields are projected onto the outer edges of finite difference cells at time t+dt, they can be used along with the other $\bar{e}$-field components on edges crossing the boundary and on inside edges to advance $\bar{h}$ on the boundary faces from time t-dt/2 to t+dt/2 ("leapfrogging" of $\bar{e}$ and $\bar{h}$ in time). If the procedure is efficient, we will compute essentially only the correct outward propagating field from prescribed sources.

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